Radial-Basis Function Networks

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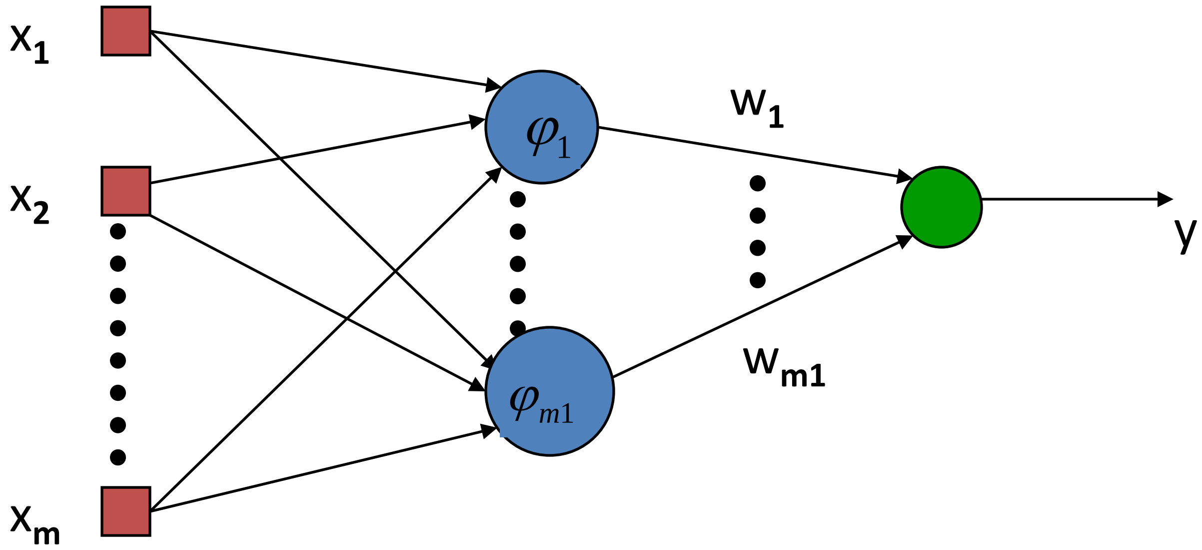
Radial-Basis Function network (RBF) is a three layers architecture:

- Input Layer: The input layer is made up of source nodes that connect the network to its environment.
- Hidden Layer: In this layer a nonlinear transformation is applied from the input space to the hidden space.
- Output Layer: The output layer is linear, supplying the response of the network to the activation pattern applied to the input layer.

RBF ARCHITECTURE



- **One hidden layer with RBF activation functions**

$$\varphi_1 \dots \varphi_{m1}$$

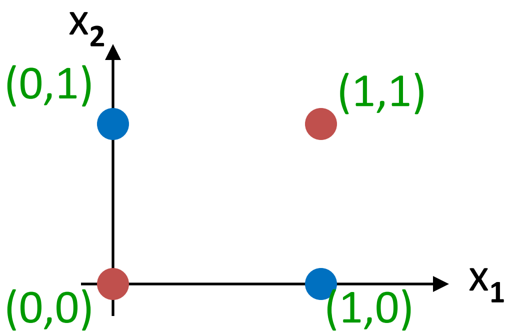
- **Output layer with linear activation function.**

$$y = w_1\varphi_1(\| x - t_1 \|) + \ldots + w_{m1}\varphi_{m1}(\| x - t_{m1} \|)$$

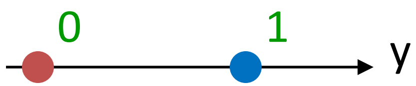
$$\| x - t \| \text{distance of } x = (x_1, \ldots, x_m) \text{ from vector } t$$

Example: XOR Problem

- Input space:



- Output space:



- Construct an RBF pattern classifier such that:

  (0,0) and (1,1) are mapped to 0, class C1

  (1,0) and (0,1) are mapped to 1, class C2

Example: the XOR problem

- In the feature (hidden layer) space:

$$\varphi_1(\| x - t_1 \|) = e^{-\|x-t_1\|^2}$$

$$\text{with } t_1 = (1,1) \text{ and } t_2 = (0,0)$$

$$\varphi_2(\| x - t_2 \|) = e^{-\|x-t_2\|^2}$$

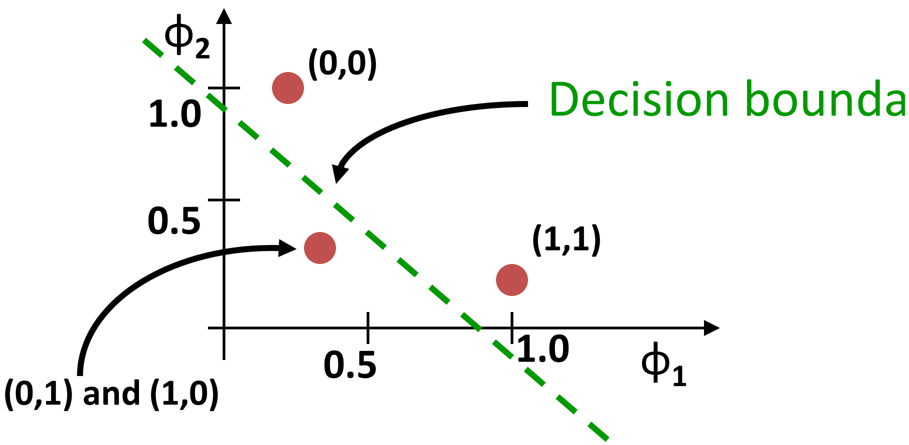


TABLE 5.1 Specification of the Hidden Functions for the XOR Problem of Example 5.1

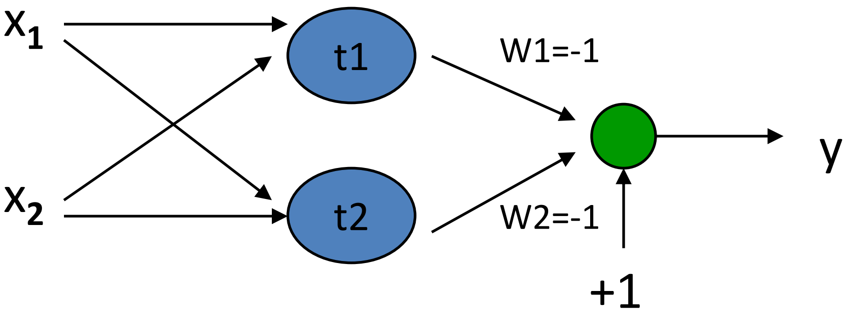
Input Pattern, $\mathbf{x}$	First Hidden Function, $\varphi_1(\mathbf{x})$	Second Hidden Function, $\varphi_2(\mathbf{x})$
(1,1)	1	0.1353
(0,1)	0.3678	0.3678
(0,0)	0.1353	1
(1,0)	0.3678	0.3678

- When mapped into the feature space < φ_1 , φ_2 > (hidden layer), C1 and C2 become *linearly separable*. So a linear classifier with $\varphi_1(x)$ and $\varphi_2(x)$ as inputs can be used to solve the XOR problem.

RBF NN for the XOR problem

$$\varphi_1(\| x - t_1 \|) = e^{-\|x-t_1\|^2} \qquad \text{with } t_1 = (1,1) \text{ and } t_2 = (0,0)$$

$$\varphi_2(\| x - t_2 \|) = e^{-\|x-t_2\|^2}$$



$$y = -e^{-\|x-t_1\|^2} - e^{-\|x-t_2\|^2} + 1$$

If $y > 0$ then class 1 otherwise class 0

Generalized Radial-Basis Function Networks

- When x_i , i=1..N is large, the one-to-one correspondence between the training input data and the Green's function produces a regularisation network that may be considered expensive.
- An approximation of the regularized network.

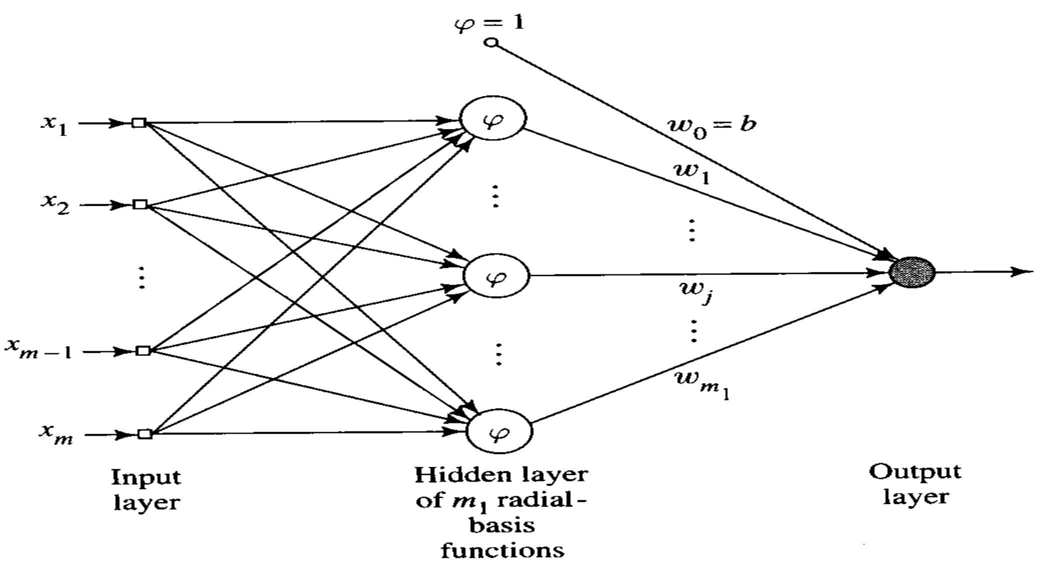


FIGURE 5.5 Radial-basis function network.

Example: XOR Problem

Table: Input output Transformation Computed for XOR problem		
Data point, j	Input pattern, X_j	Desired output, d_j
1	(1, 1)	0
2	(0,1)	1
3	(0,0)	0
4	(1,0)	1

Where G is green's function defined as:

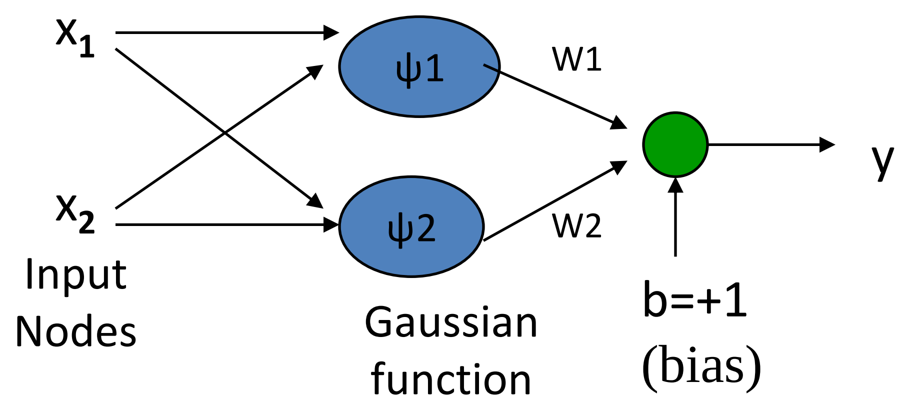
$$G = \begin{bmatrix} G(X_{1,}t_1) & G(X_{1,}t_2) & \dots.G(X_{1,}t_m) \\ G(X_{2,}t_1) & G(X_{2,}t_2) & \dots.G(X_{2,}t_m) \\ G(X_{3,}t_1) & G(X_{3,}t_2) & \dots.G(X_{3,}t_m) \\ G(X_{N,}t_1) & G(X_{N,}t_2) & \dots.G(X_{N,}t_m) \end{bmatrix} \quad G = \begin{bmatrix} 1 & 0.1353 & 1 \\ 0.3678 & 0.3678 & 1 \\ 0.1353 & 1 & 1 \\ 0.3678 & 0.3678 & 1 \end{bmatrix}$$

Calculated as $G(||\mathbf{x} - \mathbf{t}_i||) = \exp(-(\| x - ti \|)^2)$ i=1,2

Example: XOR Problem

$d=[0 \ \ 1 \ \ 0 \ \ 1]^T$ $W= [w1 \ \ w2 \ \ b]^T$

Where $W = G^+ d = (G^T G)^{-1} G^T d$ And G^+ is a pseudo inverse of G



Thus the final input output relation of the network is defined as:

$$y(x) = \sum_{i=1}^{2} wG(||x - t_i||) + b$$